

BRAIN COMPUTER INTERFACE.

Feedback effect analysis by comparison of discrimination capability of On-line and Off-line experimental procedures based on LDA.

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Abstract: This paper analyzes the user's feedback influence in the mental task discrimination capability through the comparison of results obtained from Off-line and On-line Brain Computer Interface experimental procedures. Experiments performed under these two paradigms were carried out by five male volunteers. In order to develop a wearable BCI device only two electrodes in C3 and C4 zones have been used for electroencephalographic signal acquisition. These procedures apply seven different types of preprocessing windows and Linear Discrimination Analysis technique to reduce the dimension of the feature space before the quantification of the discrimination capability between the proposed mental activities. The discrimination capability is quantified through statistical analysis, based on bilateral contrast test, between the population of the LDA transformed feature vectors. As conclusions from the results of these experiments are:

1. The user's feedback influence provokes a lower discrimination capability, but enough to be used in an On-line BCI device.
2. The use of LDA technique allows to reduce the dimensionality of the input feature space, meanwhile it is maintained the discrimination capability between the proposed mental tasks.
3. The Tukey's and rectangular preprocessing windows improve the discrimination capability.

1 Introduction.

The objective of Brain Computer Interface technology is the direct communication of user's mind with external devices, it uses the encephalographic signal as primary source of commands for the external devices (Wolpaw, J.R.; et al., 2000)(Wolpaw et al., 2002)(Birbaumer, N; et al., 2000)(Wolpaw, 2007); in the first international meeting for BCI technology, organized by the BCI group at the Wadsworth Center (New York), celebrated in Rensselaerville in 1999, it was established that BCI *"must not depend on the brain's normal output pathways of peripheral nerves and muscles"*.

A variety of methods for monitoring brain activity might serve in BCI technology: electroencephalography (EEG), magnetoencephalography (MEG), positron emission tomography (PET), functional magnetic resonance imaging (fMRI), and optical imaging. At present, only EEG meets the requirements of short time constant, affordable cost, and it is relatively simple to implement; other meth-

ods, like for example MEG or fMRI, are still technically demanding, or they have long time constants and they are less amenable to fast communication, like for example PET or optical imaging. Though EEG is the most common method for monitoring, the basic principles of BCI design and operation apply to all BCIs that use other methods to monitor brain activity.

In order to control an external device using thoughts, it is necessary to associate some mental patterns to device commands, so an algorithm that detects, acquires, filters and classifies the human electroencephalographic signal is required (Wolpaw et al., 2002)(Vidal, 1973)(Kostov, A.; Polak, M., 2000) (Pfurtscheller et al., 2000). Usually all BCI systems are compounded of the following parts:

- *Signal acquisition.* BCI devices can be categorized by the different approaches they use for the signal acquisition stage: non-invasive recordings with standard scalp electrodes, and invasive recording with epidural, subdural, or intracortical electrodes. They can also be categorized by whether they use evoked or spontaneous inputs.

In the signal-acquisition phase of BCI operation, the chosen input is acquired by the recording electrodes, amplified, and digitalized.

- *Signal processing: feature extraction.* The digitalized signals are subjected to feature extraction procedures, such as spatial filtering, voltage amplitude measurements, spectral analysis, or single-neuron separation. This analysis extracts the signal features that encode the user's messages or commands. BCIs can use signal features that are in the time domain (e.g. evoked potential amplitudes or neuronal firing rates) or the frequency domain (e.g. mu or beta-rhythm amplitudes) (Lopes da Silva, 1999). A BCI could conceivably use both time-domain and frequency-domain signal features, and might thereby improve performance. It is also possible for a BCI to use signal features, like sets of autoregressive parameters, that correlate with the user's intent but not necessarily reflect specific brain events, in those cases it is necessary to ensure that the chosen features are not contaminated by EMG, EOG or other non-CNS artifacts.
- *Signal processing: the translation algorithm.* It translates the signal features into device commands-orders that carry out the user's intent. This algorithm might use linear methods (e.g. classical statistical analysis) or nonlinear methods (e.g. neural networks).
- *The output device.* Generally the output device is a computer screen and the output is the selection of targets, letters, or icons presented on it. Initial studies are also exploring BCI control of a neuroprosthesis or orthosis that provides hand closure to people with cervical spinal cord injuries (Pfurtscheller et al.,).
- *The operating protocol.* It is the protocol that guides the operation of the BCI device. It defines how the system is turned on and off, whether communication is continuous or discontinuous, or if the message transmission is triggered by the system or by the user, the sequence and speed of interactions between user and system, and what feedback is provided to the user.

BCI devices fall into two classes: dependent and independent (Chiappa, 2006). A dependent BCI does not use the brain's normal output pathways to carry the message, but activity in these pathways is needed to generate the brain activity that does carry it. It is an alternative method for detecting messages carried in the brain's normal output pathways (e.g. gaze direction is detected by monitoring EEG rather than by monitoring eye position directly). An independent

BCI does not depend in any way on the brain's normal output pathways. The message is not carried by peripheral nerves and muscles (e.g. P300 evoked potential).

This paper focus on the user's feedback influence in the discrimination capability of three different mental activities, it analyzes the applicability of LDA to BCI and how the windowing effect affects the discrimination capability of the brain proposed activities. In order to evaluate the user's feedback influence, an On-line experimental procedure is applied, and a comparison between Off-line (Martinez, J.L.; Barrientos, A., 2007) and On-line experiment results is carried out. In the experiments considered for this report a low number of scalp-electrodes has been used to capture the endogenous electroencephalographic subject's signal. In order to facilitate the use of this technology it is important to make it easy to use, cosmesis is often crucial; that is, how the system looks and how the user looks while employing it, the number of electrodes employed in these devices is a global key feature, as the fewer of electrodes used, the higher the comfort (Wolpaw et al., 2002)(Wolpaw, 2007).

Because the main changes in brain activity are associated to changes in the power amplitude of frequency bands (Wolpaw, 2007), spectrograms based on FFT are used to obtain initial feature vectors. LDA technique is used to combine these initial features in order to reduce the dimensionality of the input space (Ripley, 2000). To minimize the leakage effect seven different types of preprocess windows has been considered: rectangular, triangular, Blackman's, Hamming's, Hanning's, Kaiser's and Tukey's (Proakis and Manolakis, 1997)(Harris, 1978) (Allen and Rabiner, 1977). The evidence of statistical difference in the feature populations associated to different brain activities has been previously shown (Martinez, J.L.; Barrientos, A., 2006).

To determine the discrimination power between the proposed cerebral activities and the effect of preprocessing window, a statistical procedure of bilateral contrast test of independent populations has been used (Peña Sanchez de Rivera, 1986), the results of each contrast is both qualitative and quantitative, qualitative in order to accept or reject the null hypothesis of equality in the population of features, quantitative in order to compare the discrimination power through significance contrast level $\alpha = 1 - p = 2.5\%$.

This article is composed of the following sections:

Section 2 describes the experimental procedures.

Section 3 describes the LDA technique.

Section 4 explains the bilateral contrast test.

Section 5 and 6 presents and analyzes the results.

Section 7 is devoted to conclusions.

2 Experimental procedures.

Off-line and On-line tests were carried out on five healthy male subjects, one of them has been trained before, but the other four were novice in the use of the system.

The Off-line tests have been carried out before On-line tests in order to have data to allow the training procedure of a simple classifier. To facilitate the mental concentration on the proposed activities, the experiments were performed in a room with low level of noise and under controlled environmental conditions, all electronic equipments external to the experiment around subject were switched off to avoid electromagnetic artifacts. The subjects were sat down in front of the acquisition system monitor, at 50 cm from the screen, their hands were in a visible position, the supervisor of the experiments controlled the correct development of them. (Neuper, C.; et al., 2001)(Penny, W. D.; et al., 2000).

2.1 Procedure for Off-line experiments.

The experimental Off-line process is shown on fig.1.

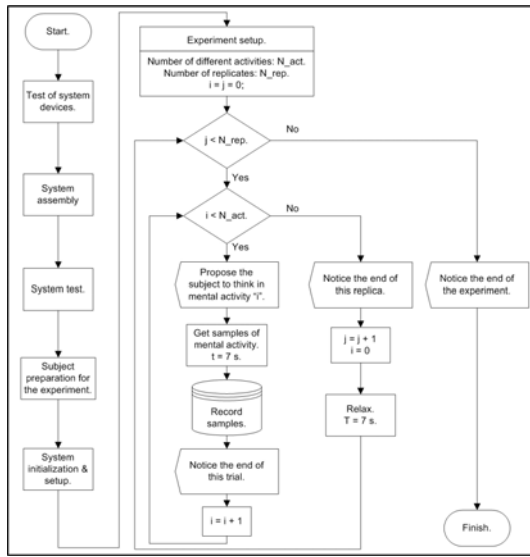


Figure 1: Diagram of the Off-line experiment realization.

Test of system devices. Checks the correct level of battery, and the correct state of the electrodes.

System assembly. Device connections: superficial electrodes (Grass Au-Cu), battery, bio-amplifier (g.BSamp by g.tec), acquisition signal card (PCI-MIO-16/E-4 by National Instrument), computer.

System test. Verifies the correct operation of the whole system. To minimize noise from the electrical network the Notch filter (50Hz) of the bio-amplifier is switched on.

Subject preparation for the experiment. Application of electrodes on subject's head. It is verified that electrode impedance was lower than 4 KOhms.

System initialization and setup. Verification of data register.

Experiment setup. The supervisor sets-up the number of replications, $N_{rep} = 10$, and the quantity of different mental activities, $N_{act} = 3$. The duration of each mental activity, a trial, is $t = 7s$, the acquisition frequency is $f_s = 384Hz$. The system randomly suggests to the subject to think about the proposed mental activity. A short relax is allowed at the end of each trial.

2.2 Procedure for On-line experiments.

The experimental On-line process is shown on fig.2. In these tests, a cursor in the center of the screen and a square goal are shown to the subject, the square goal appears half the trials on the left of the screen and the other half on the right. The subject shall try to move the cursor towards the goal thinking in the cerebral activities proposed in the Off-line experiments.

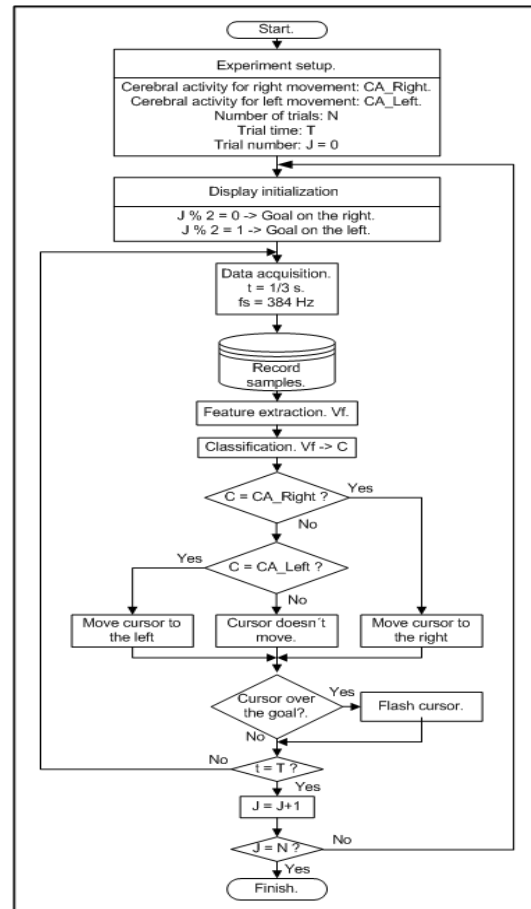


Figure 2: Diagram of the On-line experiment realization.

Experiment set-up. In this phase it is determined which cerebral activities are used to move the cursor to the left and to the right, the number of trials and the time for each trial.

Display initialization. It initializes the display, for even trials the goal is shown on the right, for odd trials on the left.

Data acquisition. In this phase 128 samples per channel are acquired at $f_s = 384\text{Hz}$.

Record samples. The previous samples are recorded for a posterior analysis.

Feature extraction. A vector of features is extracted from the acquired samples.

Classification. The vector of features is classified as belonging to one of the previous cerebral activities, and the associated movement is performed; if the vector can't be classified in any of the cerebral activities, the cursor doesn't move. If the trial time is exceeded a new trial is carried out until the N trials had been performed.

2.3 Position of the electrodes and description of cerebral activities.

For both types of experimental procedures, the electrodes were placed in the central zone of the skull, next to C3 and C4, two pair of electrodes were placed in front of and behind of Rolandic sulcus, this zone is one with the highest discriminant power, it takes signal from motor and sensory areas of the brain (Penny, W. D.; et al., 2000)(Pfurtscheller et al., 2000). Reference electrode was placed on the right mastoid, two more electrode are placed near to the corner of the eyes to register blinking.



Figure 3: Electrode placement.

The supervisor of the experiment asks the subject to figure out the following mental activities, these activities will be the cerebral patterns or tasks to differentiate among them.

Activity A. Mathematical task. Recursive subtraction of a prime number from a big quantity.

Activity B. Motor imagery. The subject imagines moving their limbs or hands, but without the materialization of the movement.

Activity C. Relax. The subject is relaxed.

2.4 Feature selection.

For Off-line experiments the registered signal is chopped in packages of samples, similar to the bun-

dles of samples obtained from the acquisition card in the On-line cases. Each package has 128 samples, acquired at $f_s = 384\text{Hz}$. A vector of six features is extracted from each package, see table 1, this vector is made up as the mean of the amplitudes of the frequency bands (the FFT is used) (Proakis and Manolakis, 1997)(Neuper, C.; et al., 2001) (Martinez, J.L.; Barrientos, A., 2006)(Penny, W. D.; et al., 2000).

Because the frequency of normal human brain is under 40-50Hz, only frequencies between 6 and 38Hz have been considered.

Table 1: Feature vector.

Index	Denomination.	Frequency (Hz).
1	$\theta.$	6 - 8
2	$\alpha_1.$	9 - 11
3	$\alpha_2.$	12 - 14
4	$\beta_1.$	15 - 20
5	$\beta_2.$	21 - 29
6	$\beta_3.$	30 - 38

3 Linear Discriminant Analysis procedure.

3.1 Introduction.

Supposed C classes of observations, Linear Discriminant Analysis is a preprocess technique that finds the transformation matrix W which separates in an optimal way two or more classes. It is used in machine learning as linear classifier or as a technique to reduce the feature space dimension before the classification process. LDA considers maximizing the following objective:

$$J(W) = \frac{W^T S_B W}{W^T S_W W} \quad (1)$$

where S_B is the between classes scatter matrix and S_W is the within classes scatter matrix, the definitions of the both matrices are:

$$S_B = \sum_c N_c (\mu_c - \bar{x})(\mu_c - \bar{x})^T \quad (2)$$

$$S_W = \sum_c \sum_{i \in c} (x_i - \mu_c)(x_i - \mu_c)^T \quad (3)$$

$$\mu_c = \frac{1}{N_c} \sum_{i \in c} x_i \quad (4)$$

$$\bar{x} = \frac{1}{N} \sum_i x_i = \frac{1}{N} \sum_c N_c \mu_c \quad (5)$$

and N_c is the number of samples in class c .

Because J is invariant to rescalings of the vectors $W \rightarrow \alpha W$, hence it is possible to choose W such that the denominator is $W^T S_W W = 1$. So the problem

of maximizing J can be transformed to the following constrained optimization problem,

$$\min_W \quad -\frac{1}{2}W^T S_B W \quad (6)$$

$$s.t. \quad W^T S_W W = 1 \quad (7)$$

corresponding to the Lagrangian,

$$L_P = -\frac{1}{2}W^T S_B W + \frac{1}{2}\lambda(W^T S_W W - 1) \quad (8)$$

With solution (the halves are added for convenience):

$$S_B W = \lambda S_W W \Rightarrow S_B^{-1} S_B W = \lambda W \quad (9)$$

This is a generalized eigen-problem, and using the fact that S_B is symmetric positive definite and can hence be written as $S_B^{-1} S_B^{\frac{1}{2}}$, where $S_B^{\frac{1}{2}}$ is constructed from its eigenvalue decomposition as $S_B = U \Lambda U^T \rightarrow S_B^{\frac{1}{2}} = U \Lambda^{\frac{1}{2}} U^T$. Defining $V = S_B^{\frac{1}{2}} W$ it is get

$$S_B^{\frac{1}{2}} S_W^{-1} S_B^{\frac{1}{2}} V = \lambda V \quad (10)$$

this is a regular eigenvalue problem for a symmetric positive definite matrix $S_B^{\frac{1}{2}} S_W^{-1} S_B^{\frac{1}{2}}$, with solutions λ_k as eigen-values and V_k as eigen-vectors, which leads to solution:

$$W = S_B^{-\frac{1}{2}} V \quad (11)$$

Plugging the solution back into the objective $J(W)$, it is found that the desired solution which maximize the objective is the one with largest eigenvalues.

3.2 Operational procedure.

1. Samples from each mental tasks are obtained.
 X_a Mathematical Activity.
 X_b Movement imagination.
 X_c Relax.

2. Statistical definition of all populations.
 $\bar{\mu}_a = E[x_a] \quad S_a = E[(x_a - \bar{\mu}_a)(x_a - \bar{\mu}_a)^T] \quad (12)$
 $\bar{\mu}_b = E[x_b] \quad S_b = E[(x_b - \bar{\mu}_b)(x_b - \bar{\mu}_b)^T] \quad (13)$
 $\bar{\mu}_c = E[x_c] \quad S_c = E[(x_c - \bar{\mu}_c)(x_c - \bar{\mu}_c)^T] \quad (14)$

3. Calculation of the Between and Within scattering matrices (eq.2 & 3).

4. Application of the LDA optimizing criterion (eq.10).

5. Calculation of the transformation matrix, W (eq.11), formed by the eigen-vectors, V_k , which eigen-values are bigger than 1×10^{-4} ordered from high to low magnitudes.

6. Transformation of the data sets.

$$X_a \Rightarrow X'_a = W^T * X_a \quad (15)$$

$$X_b \Rightarrow X'_b = W^T * X_b \quad (16)$$

$$X_c \Rightarrow X'_c = W^T * X_c \quad (17)$$

7. For classification problems once the LDA transformations are completed, euclidean or Mahalanobis distances to the center of each class could be used to classify new vectors. The smallest value among the c distances classifies the new vector as belonging to class c .

4 Statistical analysis procedure.

Bilateral contrasts between two population are used to determine if there is statistical evidence of difference between the population of features obtained from each mental activity. Each component of the vector is considered to determine its significance and separability power. Bilateral contrast makes use of population variance, if the equality of both population variances is rejected it is necessary to apply a correction factor in the degrees of freedom. These contrasts were done for each type of filtering window.

- *Bilateral contrast to the variance ratio.*

The equality of variances is obtained with $R = 1$.

n_1 : sample size of the first population.

n_2 : sample size of the second population.

σ_1 : variance of the first population.

σ_2 : variance of the second population.

\hat{S}_1 : variance estimation of the first population.

\hat{S}_2 : variance estimation of the second population.

F = Fisher distribution.

T = Student distribution.

Null hypothesis H_o vs. alternative hypothesis H_1 .

$$H_o : \frac{\sigma_1}{\sigma_2} = R \quad \text{vs.} \quad H_1 : \frac{\sigma_1}{\sigma_2} \neq R \quad (18)$$

Considering that:

$$\frac{(n_1 - 1)\hat{S}_1}{\sigma_1} \rightsquigarrow \chi^2_{n_1 - 1} \quad \frac{(n_2 - 1)\hat{S}_2}{\sigma_2} \rightsquigarrow \chi^2_{n_2 - 1} \quad (19)$$

$$\frac{\frac{1}{n_1 - 1} \frac{(n_1 - 1)\hat{S}_1}{\sigma_1}}{\frac{1}{n_2 - 1} \frac{(n_2 - 1)\hat{S}_2}{\sigma_2}} = \frac{\sigma_2}{\sigma_1} \frac{\hat{S}_1}{\hat{S}_2} \rightsquigarrow F_{n_1 - 1, n_2 - 1} \quad (20)$$

Therefore under the fulfillment of the null hypothesis:

$$F_{Exp} = \frac{1}{R} \frac{\hat{S}_1}{\hat{S}_2} \rightsquigarrow F_{n_1 - 1, n_2 - 1} \quad (21)$$

The zone of H_o acceptance is:

$$a_{teo} = F_{(n_1 - 1, n_2 - 1, 1 - \frac{\alpha}{2})} \quad (22)$$

$$b_{teo} = F_{(n_1 - 1, n_2 - 1, \frac{\alpha}{2})} \quad (23)$$

$$a_{teo} \leq F_{Exp} \leq b_{teo} \quad (24)$$

- *Bilateral contrast of two independent normal and homocedastic populations.* Null hypothesis H_o vs. alternative hypothesis H_1 .

$$H_o : \mu_1 - \mu_2 = \Delta \text{ vs. } H_1 : \mu_1 - \mu_2 \neq \Delta \quad (25)$$

The variances of the both population are equal but unknown.

$$T_{Exp} = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\hat{S}(\frac{1}{n_1} + \frac{1}{n_2})}} \quad (26)$$

In which \hat{S} is the pseudo-variance of \hat{S}_1 and \hat{S}_2

$$\hat{S} = \frac{(n_1 - 1) * \hat{S}_1 + (n_2 - 1) * \hat{S}_2}{n_1 + n_2 - 2} \quad (27)$$

The zone of H_o acceptance is:

$$T_{Teo} = t_{(n_1+n_2-2, 1-\frac{\alpha}{2})} \quad (28)$$

If $|T_{Exp}| \leq T_{Teo}$ then H_o is accepted, on the contrary H_1 is accepted and H_o is rejected.

- *Bilateral contrast of two independent normal and heterocedastic populations.* The null hypothesis H_o and alternative hypothesis are similar to the previous ones, the statistical measure is:

$$T_{Exp} = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\hat{S}_1}{n_1} + \frac{\hat{S}_2}{n_2}}} \rightsquigarrow t_f \quad (29)$$

In which f is the number of degrees of freedom calculated with the Welch's formula:

$$f = \frac{(\frac{\hat{S}_1}{n_1} + \frac{\hat{S}_2}{n_2})^2}{\frac{1}{n_1+1}(\frac{\hat{S}_1}{n_1})^2 + \frac{1}{n_2+1}(\frac{\hat{S}_2}{n_2})^2} - 2 \quad (30)$$

In this case the zone of H_o acceptance is:

$$T_{Teo} = t_{(f, 1-\frac{\alpha}{2})} \quad (31)$$

If $|T_{Exp}| \leq T_{Teo}$ then H_o is accepted, on the contrary it is assumed that the populations are different.

5 Results.

The LDA technique produces only two eigen-values bigger than $1 * 10^{-4}$ for all the experiments, this originates that only two eigen-vectors are considered in the transformation matrix, W , hence the population of the six dimension feature vectors are projected in a 2D space: X_1, X_2 . Matrices in eqs. 32 and 33 show typical experimental values for λ and W .

$$\lambda = \begin{pmatrix} 0.109 & 0 \\ 0 & 0.020 \end{pmatrix} \quad (32)$$

$$W = \begin{pmatrix} \theta & \alpha_1 & \alpha_2 & \beta_1 & \beta_2 & \beta_3 \\ -0.06 & 0.22 & 0.05 & -0.05 & 0.06 & -0.9 \\ -0.37 & 0.01 & -0.002 & -0.56 & 0.73 & 0.16 \end{pmatrix}^T \quad (33)$$

In figures 4 to 15 are represented the results of the bilateral contrast test for each transformed coordinate considering the Off-line and On-line experiments. The figures show for each channel (C3'-C3'' and C4'-C4''), and for each type of preprocessing window, the results p of the associated probability of the bilateral contrast tests between the mental tasks. In order to represent the dispersion of the results the mode value and bars from 15th to 85th percentile have been used.

6 Discussion.

In this section is analyzed the user's feedback effect over the discrimination capability of the proposed mental tasks. The comparisons between the discrimination capabilities of On-line and Off-line experimental procedures are shown in the figures 4 to 15.

From the bilateral contrast test carried out with a significant level of $\alpha = 2.5\%$, $\alpha = 1 - p$, represented in figures 4 to 9 for X_1 , it is obtained that in almost all cases the null hypothesis H_o , which maintains the equality in the populations of the features associated to the mental tasks, shall be rejected for both types of experiments; it is observed that p values obtained in the bilateral contrast test of mathematical task versus motor imagery, are lower for the On-line case in both channels and with all types of preprocessing windows than the p values obtained for the Off-line case; the dispersion of the results is similar in Off-line and On-line experiments. The same analysis for X_2 , figures 10 to 15, shows that the difference rarely appears for Off-line experiments, and in any case for in the On-line cases, $p < 0.975$.

It is also shown that for X_1 , channel C4'-C4'' performs better than C3'-C3''.

On average, for both types of experiments, all preprocessing windows show statistical difference between mental tasks; the best results, with higher quantities for the mode values and lower dispersion, are obtained for X_1 with Tukey's and Kaiser's preprocessing windows. From the numerical results is observed that as higher the eigen-value magnitude, case of X_1 , the higher the value of one component of the eigen-vector, normally in β frequency band, by the contrary, as lower the eigen-value more the contribution of the rest of eigen-vector components.

The highest contrast power is obtained in the comparison of *Motor imagery* vs. *Relax*, it is followed by *Mathematical task* vs. *Relax*, and the lowest is for *Mathematical task* vs. *Motor imagery*.

In all cases only two eigen-values have got significant magnitudes, so only two eigen-vectors have been considered in the transformation matrix. This

causes that LDA technique had projected the original six dimensional feature space over a bidimensional space, weighting the power amplitude of the frequency bands and maintaining the intrinsic characteristics of each cerebral activity.

From the numerical results it is observed that the presence of artifacts is higher in the On-line experiments than in the Off-line.

7 Conclusions.

This paper analyzes the user's feedback influence in the discrimination capability of the proposed mental activities in order to be applied to an On-line BCI device. It has been statistically proven that through the use of LDA technique it is possible to reduce the dimensionality of the original input feature space, meanwhile the discrimination capability between the proposed mental tasks is maintained, allowing the control of external devices through the association of these tasks to device commands.

In this paper two experimental methodologies have been presented, an Off-line procedure aimed to the data acquisition of the user's mental tasks, and an On-line procedure in which the user had feedback about the performing of them.

From experiment results carried out by five volunteers under these two methodologies, it is possible to conclude that the user's feedback influence provokes a lower discrimination capability, but enough to be used in an On-line BCI device, (Pineda, J.A. et al., 2003).

It is also shown that Tukey's and rectangular preprocessing windows improve the discrimination capability between the considered mental tasks.

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APPENDIX

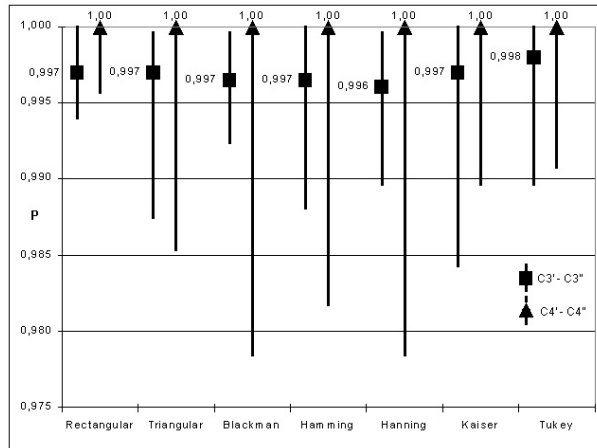


Figure 4: Off-line. Math task vs. Motor imagery. Coordinate X1.

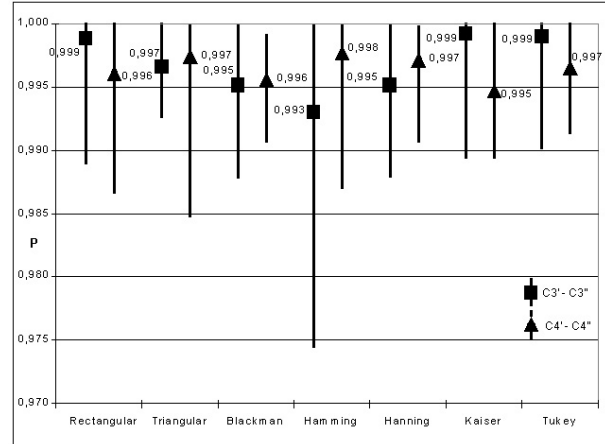


Figure 7: On-line. Math task vs. Motor imagery. Coordinate X1.

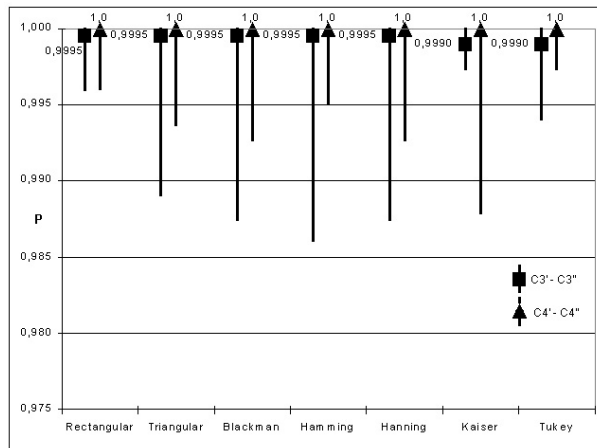


Figure 5: Off-line. Math task vs. Relax. Coordinate X1.

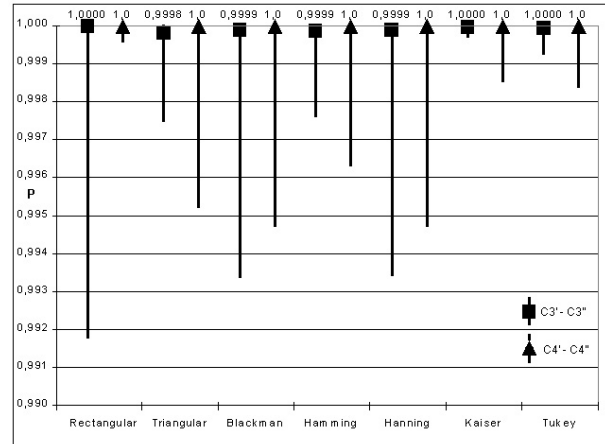


Figure 8: On-line. Math task vs. Relax. Coordinate X1.

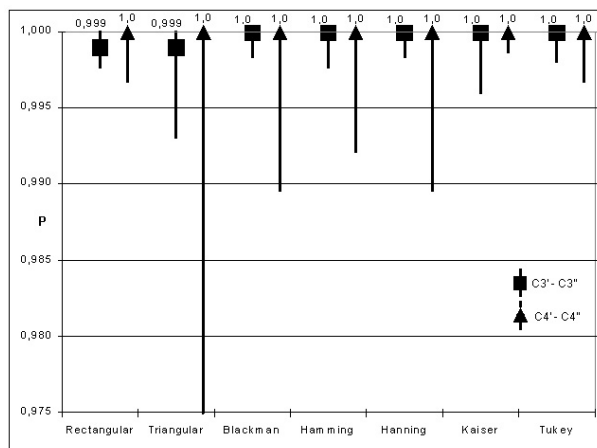


Figure 6: Off-line. Motor imagery vs. Relax. Coordinate X1.

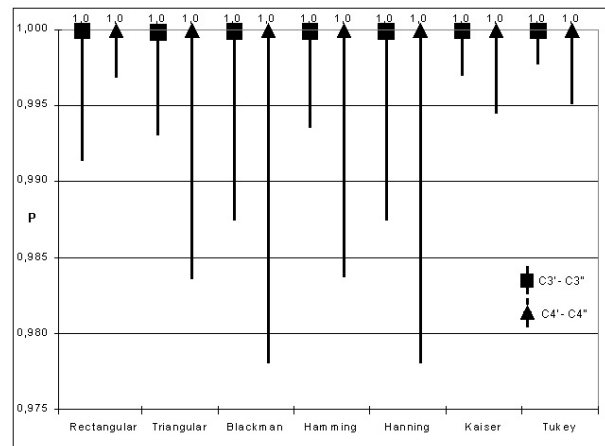


Figure 9: On-line. Motor imagery vs. Relax. Coordinate X1.

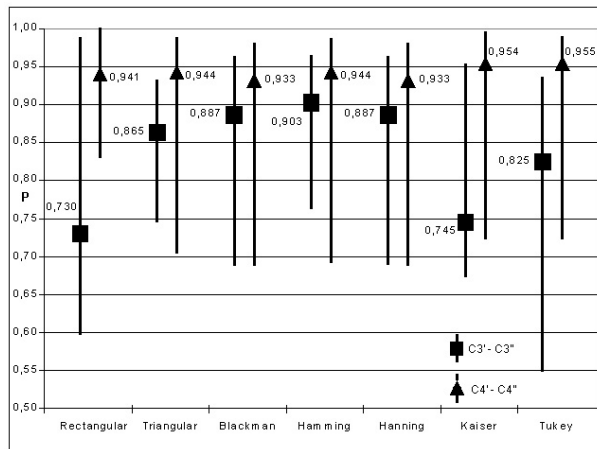


Figure 10: Off-line. Math task vs. Motor imagery. Coordinate X2.

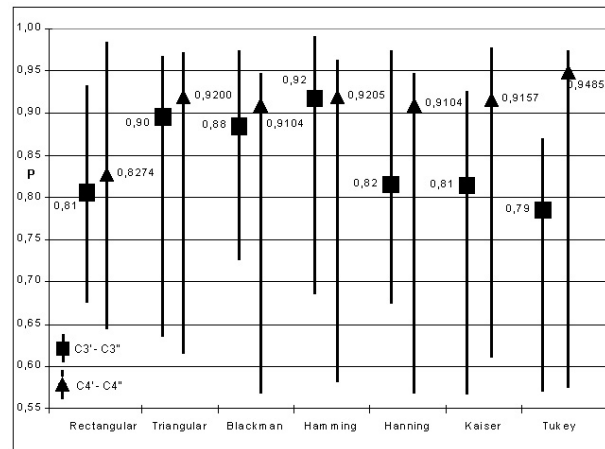


Figure 13: On-line. Math task vs. Motor imagery. Coordinate X2.

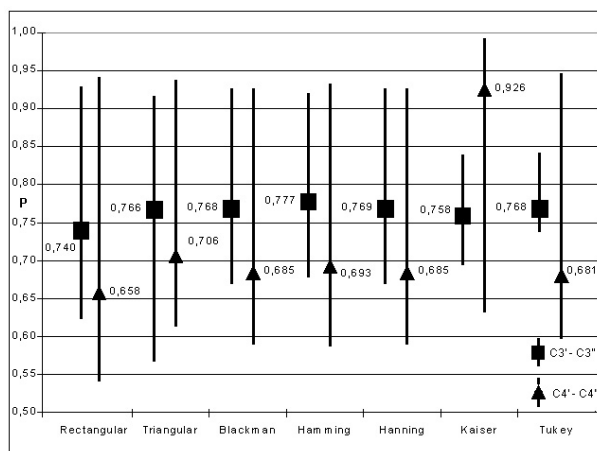


Figure 11: Off-line. Math task vs. Relax. Coordinate X2.

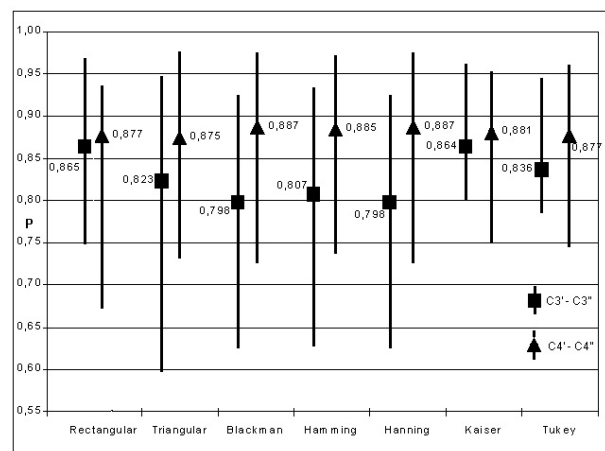


Figure 14: On-line. Math task vs. Relax. Coordinate X2.

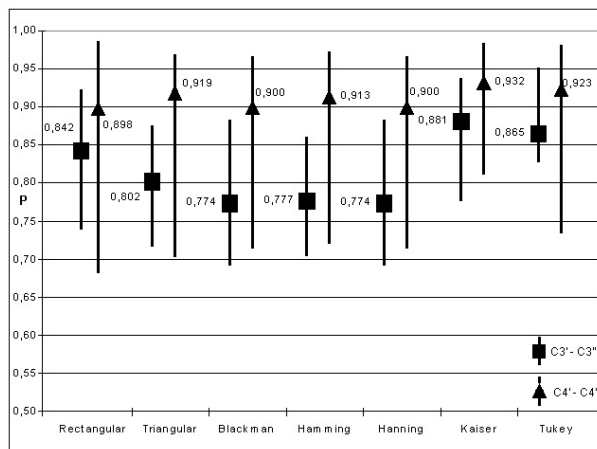


Figure 12: Off-line. Motor imagery vs. Relax. Coordinate X2

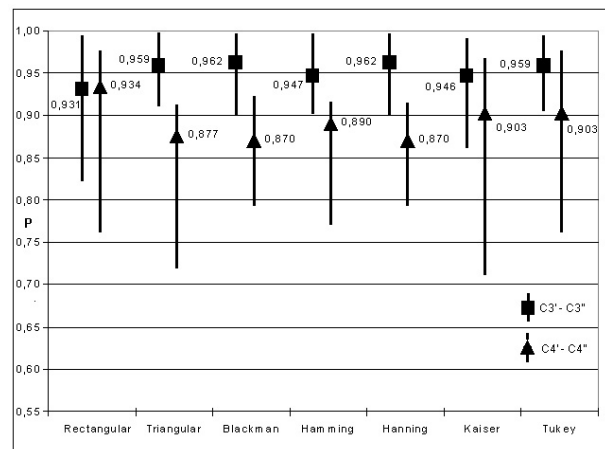


Figure 15: On-line. Motor imagery vs. Relax. Coordinate X2.